# Non-Antipodal Signaling Based Robust Detection for Cooperative Spectrum Sensing in MIMO Cognitive Radio Networks

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Abstract—In this paper we present novel detection schemes for non-antipodal signaling based cooperative spectrum sensing in multiple-input multiple-output (MIMO) cognitive radio (CR) networks, which are robust against the uncertainty in channel estimates. We consider a scenario in which the secondary users (SU) cooperate by reporting the sensed data to the fusion center for soft combining towards primary user (PU) detection. We formulate this problem employing the optimal linear discriminant and model the uncertainties in the channel state information (CSI) as ellipsoidal uncertainty sets. It is then demonstrated that this problem of PU detection with uncertainty in the channel estimates for cooperative spectrum sensing in a CR system can be formulated as second order cone program (SOCP). Further, we extend this paradigm to the associated relaxed robust detector (RRD) and multicriterion robust detector (MRD) that maximally separate the hypothesis ellipsoids in low signal-tonoise power (SNR) and deep fade channel conditions. We present a closed form solution for the proposed robust detector for the above MIMO cooperative spectrum sensing scenario. Simulation results demonstrate a significant improvement in the detection performance of the proposed uncertainty aware robust detection schemes in comparison to the conventional uncertainty agnostic matched filter detector for cooperative MIMO PU detection.

## Index Terms-CR, Cooperative spectrum sensing, SOCP.

## I. INTRODUCTION

▼OGNITIVE radio (CR) systems [1] enhance the efficiency of spectrum utilization by allowing a set of unlicensed/secondary users (SU), opportunistic access of the vacant spectral bands. Hence, it is imperative for the SUs in CR systems to reliably sense the wireless channel towards detection of weak primary user (PU) signals [2], thus avoiding interference to the licensed users. Several spectrum sensing techniques [3], [4] have been proposed in existing literature and these can be broadly classified as being local or cooperative in nature. It has been demonstrated that cooperative schemes result in a superior detection performance compared to local techniques since the former possess the ability to overcome the wireless impairments of shadowing, fading and hidden terminals, thus improving the sensing reliability. Amongst such cooperative schemes, soft-decision based maximal ratio combining [5] has been demonstrated to achieve the lowest detection error. However, its performance depends

critically on the accuracy of the channel state information (CSI) available. Obtaining perfect CSI in multiuser wireless communication scenario is a challenging task due to the time varying nature of the wireless channel. Hence, optimistically, it is only possible to obtain nominal channel estimates in practical wireless systems.

In this context we present a class of optimal detectors for non-antipodal signaling based multiple-input multiple-output (MIMO) cooperative spectrum sensing scenarios considering uncertainty in the available channel estimates. We model the inaccuracies in the channel coefficients as ellipsoidal uncertainty sets centered at the nominal channel estimates. It is demonstrated that the problem of optimal PU detection can be formulated as a second order cone program (SOCP). We describe a closed form solution for the proposed robust detector. Subsequently we also present the allied relaxed robust detector (RRD) and multicriterion robust detector (MRD) for PU detection in adverse deep fade and CSI uncertainty scenarios. Simulation results demonstrate that the proposed robust cooperative detectors have a significantly superior performance compared to the conventional matched filter (MF) detector.

The rest of the paper is organized as follows. Section II describes the system model for cooperative spectrum sensing in MIMO CR wireless networks followed by the uncertainty model for the channel estimates. In Section III we formulate the proposed robust detectors for cooperative spectrum sensing in CR networks. Simulation results are presented in Section IV and we conclude with Section V.

### **II. SYSTEM MODEL**

We consider a CR network consisting of a PU base-station with  $N_t$  transmit antennas and N cooperating SUs with each SU having  $N_r$  receive antennas. The baseband system model of the wireless multiuser MIMO CR system for the  $k^{\text{th}}$ transmitted symbol vector is given as,

$$\mathbf{y}_{i}(k) = \mathbf{H}_{i}\mathbf{x}\left(k\right) + \boldsymbol{\eta}_{i}\left(k\right), \tag{1}$$

where  $\mathbf{y}_i(k) \in \mathbb{C}^{N_r \times 1}$  is the received signal vector at the  $i^{\text{th}}$  SU corresponding to the PU base-station broadcast beacon vector  $\mathbf{x}(k) \in \mathbb{C}^{N_t \times 1}$ . The additive white Gaussian noise (AWGN) vector  $\boldsymbol{\eta}_i(k) \in \mathbb{C}^{N_r \times 1}$  of  $i^{\text{th}}$  SU at time instant k has covariance  $\mathbb{E}\left\{\boldsymbol{\eta}_i(k) \boldsymbol{\eta}_i(k)^H\right\} = \sigma^2 \mathbf{I}_{N_r}$ . Each complex element  $h_i(r,t)$  of the MIMO channel matrix  $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$  denotes the fading channel coefficient between the  $t^{\text{th}}$  transmit antenna of the PU base-station and the  $r^{\text{th}}$  receive antenna of

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the *i*<sup>th</sup> SU. Let the concatenated channel matrix  $\mathbb{H} \in \mathbb{C}^{NN_r \times N_t}$  corresponding to N cooperating SUs be defined as,

$$\mathbb{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_N \end{bmatrix}$$

The fusion center receives the measurements  $\mathbf{y}_i(k), 1 \le i \le N$ from the N SUs and jointly processes the collected data towards PU presence or spectral hole detection. Hence the above system model at the fusion center can be described as,

$$\mathbf{y}\left(k\right) = \mathbb{H}\mathbf{x}\left(k\right) + \boldsymbol{\eta}\left(k\right),\tag{2}$$

where  $\mathbf{y}(k) = [\mathbf{y}_1(k)^T, \dots, \mathbf{y}_N(k)^T]^T \in \mathbb{C}^{NN_r \times 1}$  denotes the concatenated fusion center signal corresponding to the PU base-station broadcast beacon signal  $\mathbf{x}(k) \in \mathbb{C}^{N_t \times 1}$  and the vector  $\boldsymbol{\eta}(k) = [\boldsymbol{\eta}_1(k)^T, \dots, \boldsymbol{\eta}_N(k)^T]^T$  denotes the concatenated receiver noise vector. Consider a scenario in which the PU base-station transmits the non-antipodal beacons  $\mathbf{p}_0, \mathbf{p}_1 \in \mathbb{C}^{N_t \times 1}$  to indicate the absence or presence of the licensed PU respectively. For example, in practical cellular scenarios  $\mathbf{p}_0=\mathbf{0}$  corresponds to the broadcast signal of the base-station. Define the vector  $\mathbf{h}_i \in \mathbb{C}^{NN_r \times 1}$ , for i=0, 1 as  $\mathbf{h}_i = \mathbb{H} \mathbf{p}_i$ . The PU detection problem can be formulated as the binary hypothesis testing problem,

$$\mathcal{H}_{0}: \mathbf{y}(k) = \mathbf{h}_{0} + \boldsymbol{\eta}(k)$$
$$\mathcal{H}_{1}: \mathbf{y}(k) = \mathbf{h}_{1} + \boldsymbol{\eta}(k), \qquad (3)$$

with the null hypothesis  $\mathcal{H}_0$  and alternative hypothesis  $\mathcal{H}_1$ denoting the absence and presence of the PU respectively. The optimal detector that minimizes the detection error for the above AWGN scenario is the standard MF detector [6] that optimally separates the two hypothesis. In this context, the recent progress in convex optimization has led to the development of powerful techniques for computation of the optimal linear discriminant that are described in detail in [7]. Thus the optimal hyperplane that separates the two competing hypothesis, defined by the normal vector w, can be formulated as the solution of the equivalent convex optimization framework towards the optimal linear discriminant computation,

minimize 
$$\|\mathbf{w}\|_2$$
  
subject to  $\mathbf{w}^H \mathbf{h}_0 + z \le -1$   
 $\mathbf{w}^H \mathbf{h}_1 + z \ge 1,$  (4)

where  $z \in \mathbb{C}$  is a constant. The above convex program presents a novel reformulation of the standard optimal binary detection problem, and yields a framework which forms the basis for the development of more sophisticated detectors in challenging scenarios employing convex optimization. In practical wireless scenarios it is frequently not possible to obtain accurate CSI due to the fast fading nature of the wireless channel coupled with the high noise floor, limited feedback and other impediments in wireless receivers. In such scenarios one can model the CSI uncertainty in the nominal channel

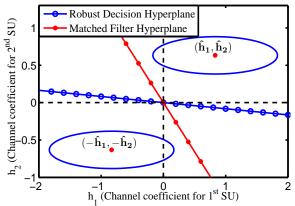


Fig. 1. Plot of the decision hyperplane with uncertainty in the SU channel coefficients, with N=2,  $N_r=1$ ,  $N_t=1$ , statistical variation matrix  $\mathbf{A}=\mathcal{D}(1, 0.25)$ , and  $h_1$  and  $h_2$  are fading channel coefficients of the cooperating users. estimate  $\hat{\mathbf{h}}$  as,

$$\mathbf{h} \in \left\{ \hat{\mathbf{h}} + \mathbf{A}\mathbf{u} \mid \|\mathbf{u}\| \le 1 \right\},\tag{5}$$

where the true channel coefficient h lies in an  $NN_r$  dimensional uncertainty ellipsoid centered at  $\hat{\mathbf{h}}$  and the vector  $\mathbf{u} \in$  $\mathbb{C}^{NN_r \times 1}$  is such that  $\|\mathbf{u}\|_2 \leq 1$ . The matrix  $\mathbf{A} \in \mathbb{C}^{NN_r \times NN_r}$ describes the statistical uncertainty in  $\mathbf{h}$ . As the N pairs of cooperating SUs can have different estimation accuracies, it is appropriate to model the channel estimation error as an ellipsoidal uncertainty set. This is a standard model employed to characterise CSI uncertainty and is described in detail in [8]. Note that this is a more general model compared to the restrictive spherical uncertainty employed in works such as [9], [10]. The framework to characterize the radius of uncertainty, i.e. matrix A for any general estimator based on the Chebyshev bounding probability is given in [11]. For example, consider the scenario shown in Fig. 1 with N=2 SUs each having  $N_r=1$  receive antenna and  $N_t=1$  transmit antenna. Considering the channel uncertainty described in (5), the true channel coefficients  $h_1, h_2$  lie in an uncertainty ellipsoid centered at the nominal channel estimates  $h_1, h_2$  corresponding to the two SUs. The statistical uncertainty matrix  $\mathbf{A} = \mathcal{D}(1, 0.25)$ , where  $\mathcal{D}$  denotes a diagonal matrix, implying that the estimate of  $h_2$ has a greater reliability compared to the estimate of  $h_1$ . Hence the optimal decision hyperplane is the one that maximally separates the two ellipsoids as illustrated in Fig. 1. Next we present a novel framework for CSI uncertainty based optimal detection in cooperative spectrum sensing scenarios.

#### III. ROBUST DETECTION WITH CSI UNCERTAINTY

Based on the above description of the adverse wireless detection scenario, the optimal robust detector can thus be naturally formulated to maximize the worst case distance between the ellipsoidal uncertainty sets corresponding to the two hypotheses. Hence the optimal decision hyperplane for the MIMO CR cooperative spectrum sensing scenario can be computed as the solution to the convex program,

minimize 
$$\|\mathbf{w}\|_2$$
  
subject to  $\max_{\|\mathbf{u}\| \le 1} \mathbf{w}^H (\hat{\mathbf{h}}_0 + \mathbf{A}\mathbf{u}) + z \le -1$  (6)  
 $\min_{\|\mathbf{u}\| \le 1} \mathbf{w}^H (\hat{\mathbf{h}}_1 + \mathbf{A}\mathbf{u}) + z \ge 1,$  (7)

where it can be seen that the constraints (6) and (7) in the above optimization problem denote the worst case ellipsoidal distance. Thus the optimal robust separating hyperplane maximizes the worst case distance between the uncertainty ellipsoids and depends significantly on the directional nature of CSI reliability as shown in Fig. 1. Further, observing that  $\max_{\|\mathbf{u}\| \le 1}(\mathbf{w}^H \mathbf{A} \mathbf{u})$  occurs when  $\mathbf{u} = \frac{\mathbf{A}^H \mathbf{w}}{\|\mathbf{A}^H \mathbf{w}\|}$ , the above robust problem can be equivalently formulated as,

minimize 
$$\|\mathbf{w}\|_2$$
  
subject to  $\mathbf{w}^H \hat{\mathbf{h}}_0 + \|\mathbf{A}^H \mathbf{w}\| + z \le -1$   
 $\mathbf{w}^H \hat{\mathbf{h}}_1 - \|\mathbf{A}^H \mathbf{w}\| + z \ge 1$  (8)

This paradigm for the robust hyperplane computation in nonantipodal signalling scenarios can be readily recognized as a SOCP [7]. Further, we describe below the closed form expression for the optimal robust decision hyperplane based on the above optimization framework. Consider  $\tilde{\mathbf{y}}(k) \in \mathbb{C}^{NN_r \times 1}$ defined as  $\tilde{\mathbf{y}}(k) = \mathbf{y}(k) - \mathbb{H}\bar{\mathbf{p}}$ , where  $\bar{\mathbf{p}} = \frac{1}{2}(\mathbf{p}_0 + \mathbf{p}_1)$ . Accordingly, the equivalent received signal model at the fusion center can be described from (2) as,

$$\underbrace{\mathbf{y}\left(k\right) - \mathbb{H}\bar{\mathbf{p}}}_{\tilde{\mathbf{y}}\left(k\right)} = \mathbb{H}\left(\underbrace{\mathbf{x}\left(k\right) - \bar{\mathbf{p}}}_{\tilde{\mathbf{x}}\left(k\right)} + \boldsymbol{\eta}\left(k\right),\tag{9}$$

where  $\tilde{\mathbf{x}}(k) = \mathbf{x}(k) - \bar{\mathbf{p}}$ . The hypothesis points for the above modified detection paradigm correspond to the vectors  $\pm \mathbf{h}$  defined as  $\mathbf{h} = \frac{1}{2} \mathbb{H}(\mathbf{p}_1 - \mathbf{p}_0)$ . Thus the framework for the optimal robust detection described in (8) equivalently reduces to,

minimize 
$$\|\mathbf{w}\|_2$$
  
subject to  $-\mathbf{w}^H \hat{\mathbf{h}} + \|\mathbf{A}^H \mathbf{w}\| + z \le -1$   
 $\mathbf{w}^H \hat{\mathbf{h}} - \|\mathbf{A}^H \mathbf{w}\| + z \ge 1.$  (10)

From (9) it can be observed that the two  $NN_r$  dimensional hypothesis vectors are antipodal. Hence, the decision hyperplane that optimally separates the two hypothesis vector is homogeneous, i.e. z = 0 in (10). Thus the two constraints in the above framework are identical. Hence the above optimization problem for robust cooperative spectrum sensing can be recast as the equivalent symmetric SOCP,

minimize 
$$\|\mathbf{w}\|_2$$
  
subject to  $\mathbf{w}^H \hat{\mathbf{h}} - \|\mathbf{A}^H \mathbf{w}\| \ge 1.$  (11)

The standard lagrangian cost function  $L(\mathbf{w}, \mu)$  for the above optimization problem can be formulated as,

$$L(\mathbf{w},\mu) = \|\mathbf{w}\|^{2} + \mu \left( \|\mathbf{A}^{H}\mathbf{w}\|^{2} - (\mathbf{w}^{H}\hat{\mathbf{h}} - 1)^{2} \right)$$
  
$$= \mathbf{w}^{H} \left(\mathbf{I} + \mu \mathbf{P}\right) \mathbf{w} + 2\mu \mathbf{w}^{H}\hat{\mathbf{h}} - \mu, \qquad (12)$$

where  $\mathbf{P} = \mathbf{A}\mathbf{A}^H - \hat{\mathbf{h}}\hat{\mathbf{h}}^H$ . As demonstrated in [8], from the KKT conditions for the above convex optimization problem, the optimal value of the Lagrange multiplier  $\mu_{\text{opt}}$  can be computed as the zero of the scalar secular equation given as,

$$f(\mu) = \mu^2 \sum_{i=1}^{NN_r} \frac{\hat{h}_i^2 p_i}{(1+\mu p_i)^2} - 2\mu \sum_{i=1}^{NN_r} \frac{\hat{h}_i^2}{(1+\mu p_i)} - 1, \quad (13)$$

where  $p_i \in \mathbb{R}^n$  are the diagonal elements of **P** and the vector  $\hat{\mathbf{h}} = [\hat{h}_1^T, \hat{h}_2^T, \dots, \hat{h}_{NN_r}^T]^T$ . On computing the Lagrange multiplier  $\mu_{\text{opt}}$  that satisfies  $f(\mu_{\text{opt}})=0$ , the optimum  $\mathbf{w}_{\text{opt}}$  corresponding to the robust hyperplane that maximally separates the ellipsoidal uncertainty sets can be derived as,

$$\mathbf{w}_{\text{opt}} = -\mu_{\text{opt}} (\mathbf{I} + \mu_{\text{opt}} \mathbf{P})^{-1} \mathbf{h}.$$
 (14)

Next we present the allied framework of RRD and MRD.

## A. Relaxed and Multicriterion Robust Detection

In low SNR and deep fade scenarios, the hypothesis ellipsoids potentially overlap and thus can not be strictly separated by a decision hyperplane. In such scenarios one can modify the robust detection paradigm to compute the optimal hyperplane that minimizes the size of the set of misclassified points through relaxed robust discrimination (RRD) as discussed in [7, Section 8.6.1], which can be formulated as,

minimize b  
subject to 
$$\mathbf{w}^{H}\hat{\mathbf{h}}_{0} + \|\mathbf{A}^{H}\mathbf{w}\| + z \leq -1 + b$$
  
 $\mathbf{w}^{H}\hat{\mathbf{h}}_{1} - \|\mathbf{A}^{H}\mathbf{w}\| + z \geq 1 - b$   
 $b \geq 0,$ 
(15)

where b, is the non-negative slack variable and denotes the measure of constraint violation. This can be readily seen as a SOCP that yields a relaxed optimal detector for cooperative spectrum sensing applications. A multicriterion robust detector (MRD), which employs a trade-off between the worst case ellipsoidal separation and the constraint violation can be formulated as,

minimize 
$$\|\mathbf{w}\|_2 + \lambda b$$
  
subject to  $\mathbf{w}^H \hat{\mathbf{h}}_0 + \|\mathbf{A}^H \mathbf{w}\| + z \le -1 + b$   
 $\mathbf{w}^H \hat{\mathbf{h}}_1 - \|\mathbf{A}^H \mathbf{w}\| + z \ge 1 - b$   
 $b \ge 0,$  (16)

where  $\lambda$  is a non-negative weighing parameter. In the next section we present simulation results to validate the performance of the proposed robust PU detection schemes.

## **IV. SIMULATION RESULTS**

We consider a 2×2 MIMO scenario, i.e. each CR user has  $N_r=2$  receive antennas and the PU base station has  $N_t=2$  transmit antennas with N=2 SUs. We consider the transmission of beacons  $\mathbf{p}_0=[0,0]^T$  and  $\mathbf{p}_1=[\sqrt{2},\sqrt{2}]^T$  corresponding to the absence and presence of PU respectively. Our simulation setup incorporates different levels of CSI uncertainty, which are characterized by the uncertainty matrices  $\mathbf{A}_i=\mathbf{U}\mathbf{D}_i\mathbf{U}^T$ , where  $\mathbf{U}$  is a random unitary matrix, and the diagonal matrix  $\mathbf{D}_i=\mathcal{D}(\mathbf{d}_i)$ , where  $\mathbf{d}_i\in\mathbb{R}^{NN_r\times 1}$ . The modelling of such ellipsoids is discussed in detail [8], [12]. In Fig. 2(a) we compare the detection error performance of the robust detector (8) for cooperative spectrum sensing with the nominal channel estimate based matched filter (MF Nominal) detector and the genie aided true channel coefficient based matched filter (MF Genie) detector considering different levels of CSI

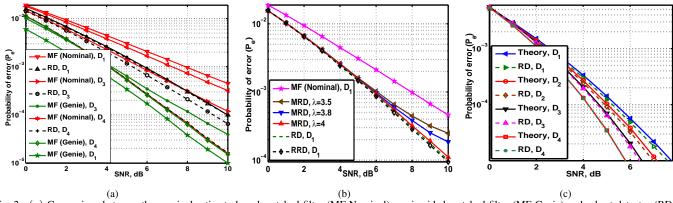


Fig. 2. (a) Comparison between the nominal estimate based matched filter (MF Nominal), genie aided matched filter (MF Genie) and robust detector (RD), (b) Comparison between MF, RD, relaxed robust detector (RRD) and multicriterion robust detector (MRD), (c) Comparison between RD and closed form solution, for  $N_r=2$ ,  $N_t=2$  MIMO, N=2,  $\mathbf{p}_0=[0,0]^T$ ,  $\mathbf{p}_1=[\sqrt{2},\sqrt{2}]^T$ ,  $\mathbf{D}_1=\mathcal{D}$  (1.6, 1.4, 1.2, 1),  $\mathbf{D}_2=\mathcal{D}$  (1.28, 1.12, 0.98, 0.8),  $\mathbf{D}_3=\mathcal{D}$  (0.8, 0.7, 0.6, 0.5) and  $\mathbf{D}_4=\mathcal{D}$  (0.32, 0.28, 0.24, 0.2).

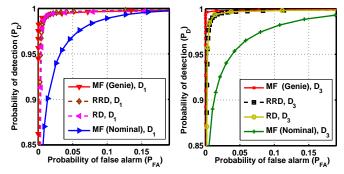


Fig. 3. Probability of detection vs. probability of false alarm comparison between the nominal estimate based matched filter (MF Nominal), genie aided matched filter (MF Genie), robust detector (RD) and relaxed robust detector (RRD) for  $N_r=2$ ,  $N_t=2$  MIMO, N=2,  $\mathbf{p}_0=[0,0]^T$ ,  $\mathbf{p}_1=[\sqrt{2},\sqrt{2}]^T$ , SNR=1 dB,  $\mathbf{D}_1=\mathcal{D}(1.6, 1.4, 1.2, 1)$  and  $\mathbf{D}_3=\mathcal{D}(0.8, 0.7, 0.6, 0.5)$ .

uncertainty. It is evident from the results that the robust detector significantly outperforms the nominal channel estimate based matched filter detector and further the performance gap between the competing detectors progressively increases with the increase in uncertainty. In Fig. 2(b) we compare the detection performance of the proposed robust detector with that of the related relaxed robust detection (RRD) and multicriterion robust detection (MRD) schemes introduced in (15) and (16) respectively. It can be seen from the figure that RRD has a performance edge over the robust detector while significantly outperforming the conventional matched filter detector. It can also be seen that the MRD has a performance similar to that of the robust detector. From Fig. 2(c) it can be observed that the performance of the robust detector in (8) employing the CVX solver [13] is in close agreement with that obtained from the closed form solution in (14). Finally in Fig. 3 we plot the probability of detection  $(P_D)$ vs. probability of false alarm  $(P_{FA})$ . From the figure it can be observed that both the robust detector and the relaxed robust detector have a superior performance compared to the nominal channel estimate based matched filter detector and it can also be observed that they are close to the performance of the true channel coefficient based genie aided matched filter detector.

## V. CONCLUSION

In this paper we have developed novel techniques for cooperative spectrum sensing in non-antipodal MIMO CR

scenarios. We proposed a robust detector for soft-decision based cooperative spectrum sensing which considers the channel uncertainty. It has been demonstrated that the worst case detection error minimization in the above scenario can be formulated as a SOCP. We further derived a closed form expression for the optimal robust detector based on reducing the above optimization problem to a symmetric SOCP. The proposed uncertainty aware robust detector and the associated RRD and MRD schemes described in this work have been demonstrated to yield superior performance compared to the conventional matched filter detector for CR spectrum sensing.

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